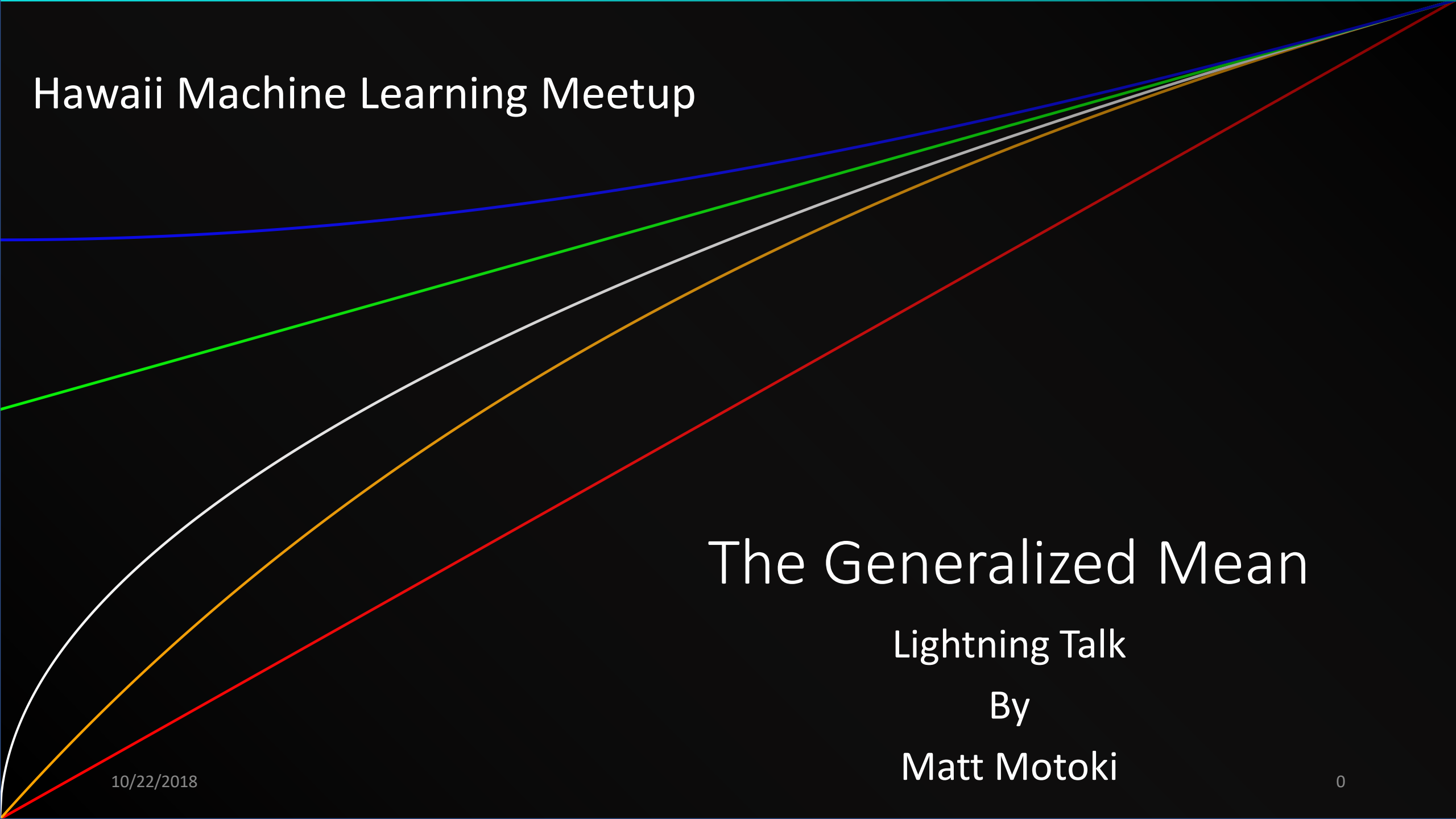
Hawaii Machine Learning Meetup



The Generalized Mean

Lightning Talk

By

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Pop Quiz

Question: What is the mean of 0, 0.5, and 1?

a) 0

b) 0.5

c) 1

d) All of the above.

Answer: d) All of the above.

What do you mean!?!?!?

Different Types of Means

Arithmetic mean:

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

Harmonic mean:

$$\bar{x} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$

Geometric mean:

$$\bar{x} = (x_1 \times x_2 \times \cdots \times x_n)^{1/n}$$

Example 1

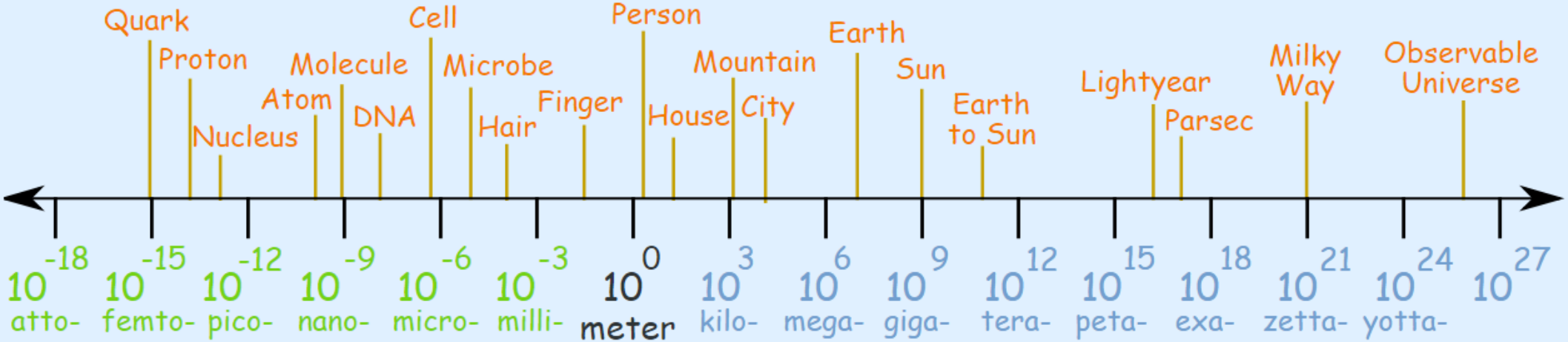
Question: “You drive at x_1 mph to Grandma’s house, and then x_2 mph back; what was your average speed?”

Answer: Assume Grandma’s house is L miles away. It takes $t_1 = L/x_1$ hours to get to Grandma’s house and $t_2 = L/x_2$ hours to get back.

The average speed is then

$$\bar{x} = \frac{2L}{t_1 + t_2} = \frac{2L}{\frac{L}{x_1} + \frac{L}{x_2}} = \frac{2}{\frac{1}{x_1} + \frac{1}{x_2}}.$$

Example 2



Question: What is the mean between

- The radius a molecule of water: $0.275 \times 10^{-9}\ \mathrm{m}$
- The radius of a Mountain Everest: $8.8 \times 10^{3}\ \mathrm{m}$

Answer: $\sqrt{(0.275 \times 10^{-9}\ \mathrm{m}) \times (8.8 \times 10^{3}\ \mathrm{m})} \approx 0.0016\ \mathrm{m}$

Example 3

Suppose you want to buy a new camera.

- One camera has a zoom of 200 percent and gets an 8 in reviews
- The other has a zoom of 300 percent and gets a 3 in reviews

The arithmetic mean gives:

- $(200 + 8)/2 = 104$
- $(300 + 3)/2 = 151.5$

The geometric mean gives:

- $\sqrt{200 \times 8} = 40$
- $\sqrt{300 \times 3} = 30$

What do they have in common?

Arithmetic Mean

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

Harmonic Mean

$$\bar{x} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$

Geometric Mean

$$\bar{x} = (x_1 \times x_2 \times \cdots \times x_n)^{1/n}$$

They are all special cases of:

$$\bar{x} = \left(\frac{1}{n} \sum_{i=1}^{n} x_i^p \right)^{1/p}$$

Arithmetic Mean: $p = 1$

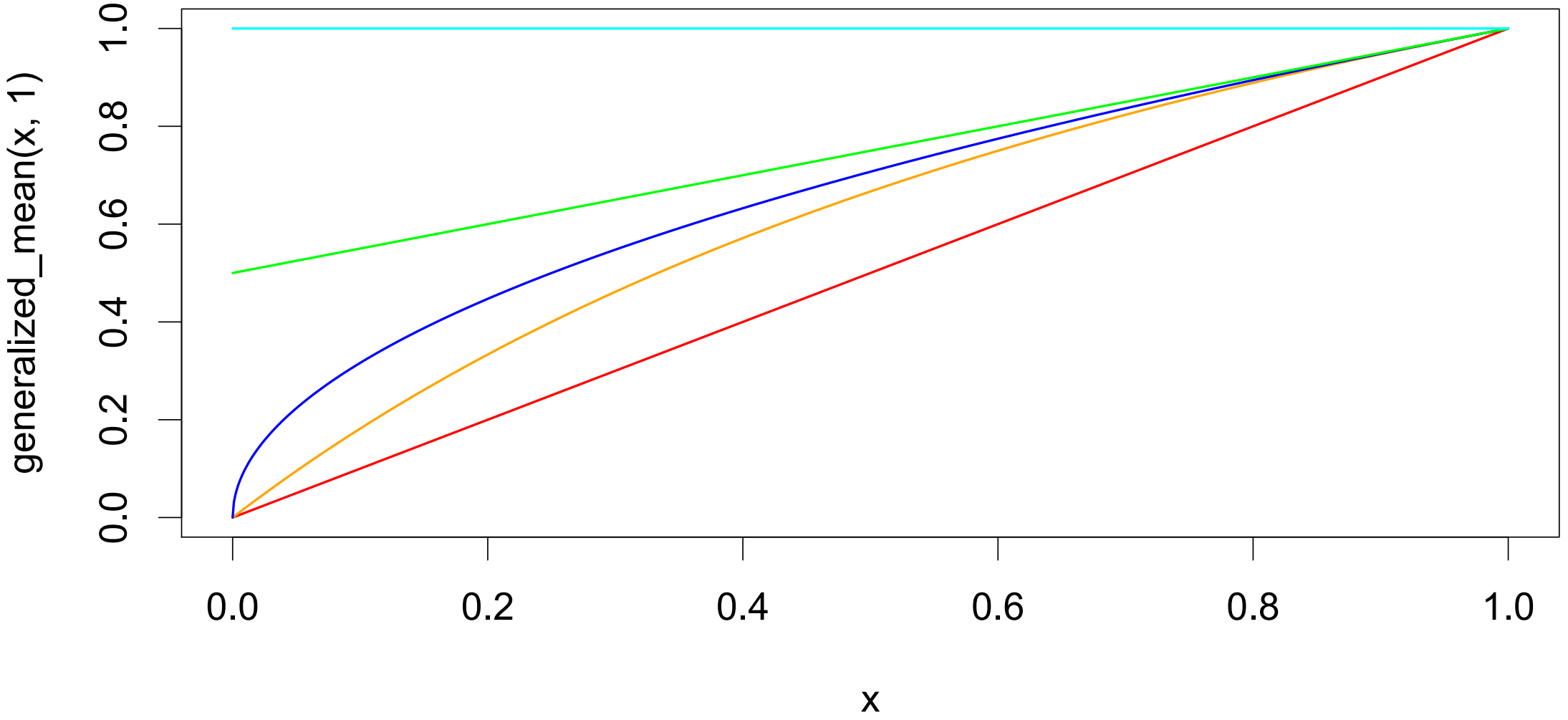
Harmonic Mean: $p = -1$

Geometric Mean: $p \to 0$

Minimum: $p \to -\infty$

Maximum: $p \to +\infty$

Generalized Mean Visualized



Pop Quiz (Revisited)

Question: What is the mean of 0, 0.5, and 1?

a) 0 $p \to -\infty$

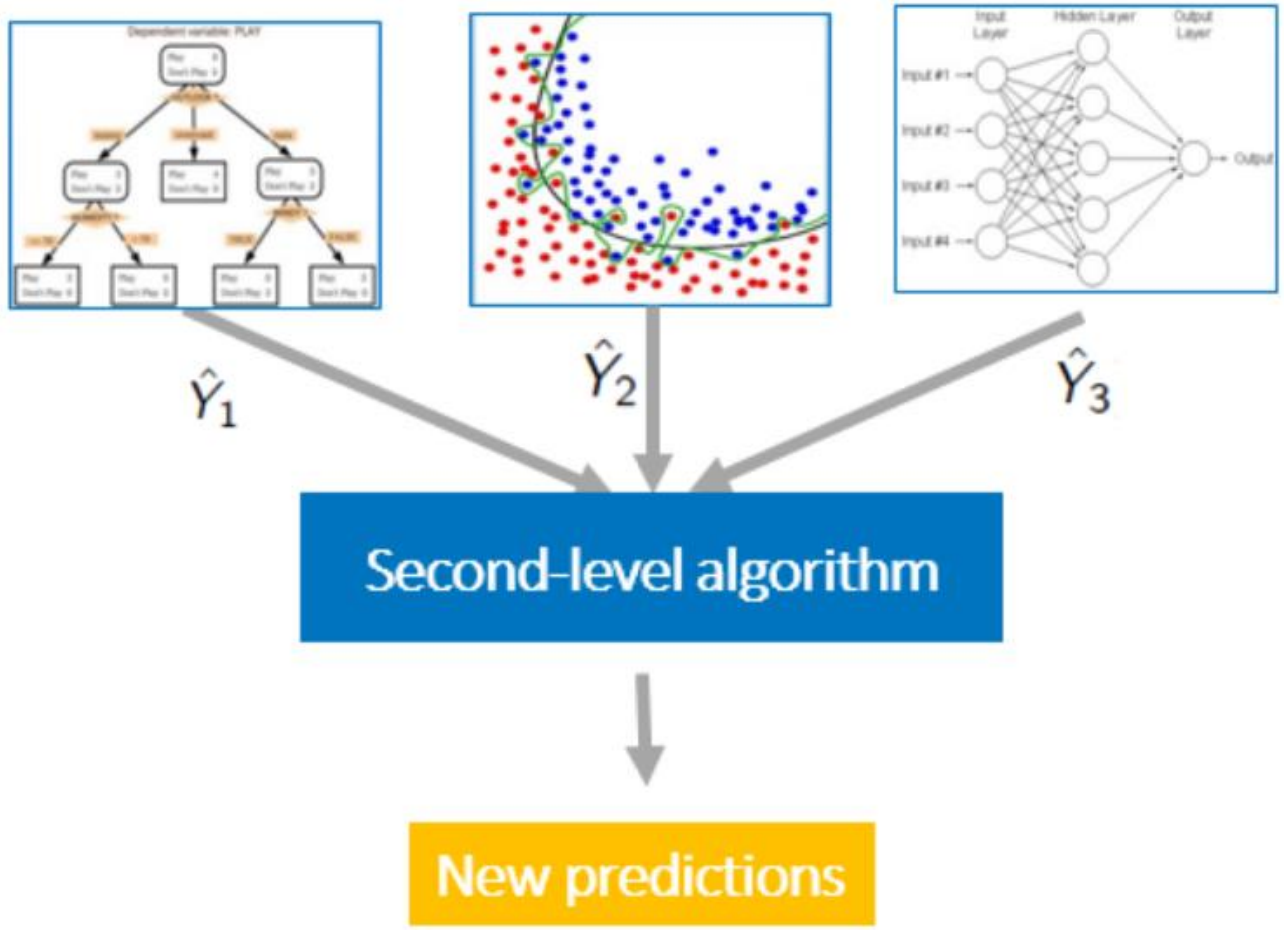
b) 0.5 $p = 1$

c) 1 $p \to +\infty$

d) All of the above.

Answer: d) All of the above.

Model Ensembling



- Ensembling combines multiple model's predictions

$$\hat{y} = f(\hat{y}_1, \hat{y}_2, \hat{y}_3)$$

- Useful technique for competitive machine learning
  - Netflix
  - ImageNet
  - Kaggle
- Averaging reduces variance

Which mean do I use?

- Use domain knowledge to make an informed decision; e.g., if we are predicting a rate, consider ensembling using the harmonic mean.
- Pay attention to the metric you are trying to optimize; e.g.,
  - When minimizing RMSE, consider ensembling with arithmetic mean
  - When minimizing RMSLE, consider ensembling with geometric mean
- Treat p as a hyperparameter

Everything is a hyperparameter – ML Wave

References

- Example 1 (Harmonic Mean) https://medium.com/@JLMC/understanding-three-simple-statistics-for-data-visualizations-2619dbb3677a
- Example 2,3 (Geometric Mean) https://www.mathsisfun.com/numbers/geometric-mean.html
- Generalized Mean http://mathworld.wolfram.com/PowerMean.html
- Model Ensembling https://mlwave.com/kaggle-ensembling-guide/